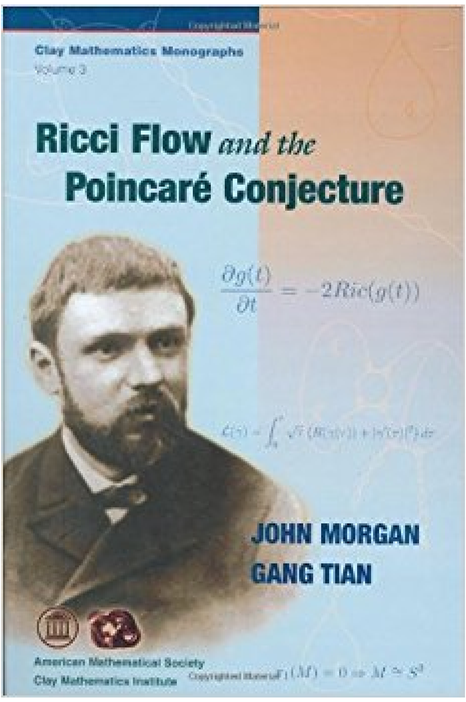
Ricci Flow And The Poincare Conjecture (Clay Mathematics Monographs)



Synopsis

For over 100 years the PoincarÃƒÂ© Conjecture, which proposes a topological characterization of the 3-sphere, has been the central question in topology. Since its formulation, it has been repeatedly attacked, without success, using various topological methods. Its importance and difficulty were highlighted when it was chosen as one of the Clay Mathematics Institute's seven Millennium Prize Problems. In 2002 and 2003 Grigory Perelman posted three preprints showing how to use geometric arguments, in particular the Ricci flow as introduced and studied by Hamilton, to establish the PoincarÃƒÂ© Conjecture in the affirmative. This book provides full details of a complete proof of the PoincarÃƒÂ© Conjecture following Perelman's three preprints. After a lengthy introduction that outlines the entire argument, the book is divided into four parts. The first part reviews necessary results from Riemannian geometry and Ricci flow, including much of Hamilton's work. The second part starts with Perelman's length function, which is used to establish crucial non-collapsing theorems. Then it discusses the classification of non-collapsed, ancient solutions to the Ricci flow equation. The third part concerns the existence of Ricci flow with surgery for all positive time and an analysis of the topological and geometric changes introduced by surgery. The last part follows Perelman's third preprint to prove that when the initial Riemannian 3-manifold has finite fundamental group, Ricci flow with surgery becomes extinct after finite time. The proofs of the PoincarÃƒÂ© Conjecture and the closely related 3-dimensional spherical space-form conjecture are then immediate. The existence of Ricci flow with surgery has application to 3-manifolds far beyond the PoincarÃƒÂ© Conjecture. It forms the heart of the proof via Ricci flow of Thurston's Geometrization Conjecture. Thurston's Geometrization Conjecture, which classifies all compact 3-manifolds, will be the subject of a follow-up article. The organization of the material in this book differs from that given by Perelman. From the beginning the authors present all analytic and geometric arguments in the context of Ricci flow with surgery. In addition, the fourth part is a much-expanded version of Perelman's third preprint; it gives the first complete and detailed proof of the finite-time extinction theorem. With the large amount of background material that is presented and the detailed versions of the central arguments, this book is suitable for all mathematicians from advanced graduate students to specialists in geometry and topology. The Clay Mathematics Institute Monograph Series publishes selected expositions of recent developments, both in emerging areas and in older subjects transformed by new insights or unifying ideas. Titles in this series are co-published with the Clay Mathematics Institute (Cambridge, MA).

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"The comprehensive and carefully detailed nature of the text makes this book an invaluable resource for any mathematician who wants to understand the technical nuts and bolts of the proof, while the introductory chapter provides an excellent conceptual overview of the entire argument." ---- Mathematical Reviews

After so much research effort was put into the PoincarÃ© conjecture for so many decades, it is a relief that it was finally solved, published, scrutinised and written up as a monograph. This book is 10 years old now. So it is no longer breaking news. But it is an excellent demonstration of how a variety of DG techniques have been combined to solve the problem.The history of Perelman’s solution for the PoincarÃ© conjecture is full of controversy, which has still not truly settled. Some accusations were made, and this book shows its bias in the title of Part 4: "Completion of the proof of the PoincarÃ© Conjecture". So beware that this is not a purely value-free dry academic book. There’s a lot of politics behind it. (See wikipedia for a quick summary.)Perhaps I should mention that the Ricci flow is quite distinct from flow by mean curvature. Ricci flow is defined on intrinsic manifolds whereas mean curvature flow is defined on an embedded or "extrinsic" manifold. I mention this because there are several books on mean curvature flow, which could easily be confused with Ricci flow. I remember discussing both kinds of flow with DG specialists in 1984, when Hamilton’s Ricci flow techniques were still new. I even did a little bit of research myself on the mean curvature flow, but as far as I know, it does not help with the Ricci flow.I think this book would be useful to anyone doing or contemplating research in the area of pure differential geometry where the objective is to constrain the combinatorial topological classes of manifolds in terms of their curvature. Anyone doing research in this area has probably read it and moved on a long time ago. For researchers peripheral to the area, this book is really good to have on the shelf as a definitive description of how the conjecture has been solved. The printing and binding are truly the best. This book should still be in excellent condition in 200 years, and still worth reading.

For those readers who sincerely want to understand the recent proposed proof of the Poincare conjecture due to the mathematician Grisha Perelman, this book is the only one so far that claims to discuss the proof in the detail required. If one is to judge a book solely by its introduction, then this one is off to a good start, for the authors give a quick overview of the strategy behind the proof along with a discussion of what background is needed to understand it. Those readers who come to the book with only a background in geometric topology will need to become familiar with various techniques and concepts in analysis and differential geometry. In the introduction the authors inform the reader that this background will be developed in later chapters, but to really understand the proof one needs an understanding of it that goes beyond the formal. The Ricci flow plays a central role in the proof, and the Ricci flow equation is presented in the introduction as a nonlinear version of the heat equation. The physicist reader will appreciate this description and perhaps wonder if it, along with the strategy of understanding the topology of manifolds that are "unions of epsilon-tubes and epsilon-caps" is best done in a quantum framework due to the its emphasis on distances and scale (the "capping" phenomena that is discussed in the introduction is somewhat reminiscent of the regularization and "smoothing" that goes on at small scales when studying quantum time evolutions or in the symplectic category the notion of capacity).Readers who insist on constructive proofs every step of the way may be disappointed to learn that the authors make frequent use of non-constructive strategies. An example of this is the proof of non-collapse where constructions are made that contradict the maximum principle of Hamilton. Another example is the establishment of the existence of canonical neighborhoods for all points of large scalar curvature. A contradiction is derived by assuming that one can find a particular sequence of points of arbitrarily large curvature where there are no canonical neighborhoods.Refreshingly, the authors do not hesitate to use diagrams in the book, increasing its didactic quality and making the mathematical constructions much easier to follow. A purely formal treatment would make the reading much more intense but no doubt readers would still draw their own diagrams if the authors chose to write the book in such a fashion. So many of modern mathematical papers and books are written this way, making their understanding much more time-consuming and making proof checking extremely difficult.Chapter one is fairly standard background in the differential geometry of Riemannian manifolds and not all proofs are given. Readers are expected to either consult the references or fill in the details of the proofs themselves if so inclined. Of particular interest is the notion of an open cone over a Riemannian manifold and how to compare curvature in Riemannian manifolds (the proof of the Bishop-Gromov relative volume comparison theorem is omitted).In chapter two the authors concentrate on the Riemannian geometry of manifolds of non-negative curvature. One of the more interesting questions in this context is to what extent a geodesic ray is minimizing, which for the case of zero curvature is illustrated by straight lines (viewed as limits of geodesics getting longer and longer). Busemann functions are introduced, which measure how far a point in a complete, noncompact, non-negative curvature manifold is out toward infinity in the direction of a geodesic ray, and the authors illustrate their most useful properties (such as Lipschitz continuity, which is crucial since distance is usually not a smooth function on Riemannian manifolds). Their definition of the Busemann function is different from some in the literature by a simple change of sign, which only has the effect of making the Busemann functions pointwise bounded below (instead of pointwise bounded above as some do). Proposition 2.3 is important, in that it shows that the Laplacian of a Busemann function is non-negative (i.e. that it is "superharmonic"). This property is used later to derive a maximum principle for complete Riemannian manifolds of non-negative Ricci curvature.Also introduced in Chapter 2 is the Toponogov theory, which allows one to compare lengths in complete Riemannian manifolds with those in Euclidean space. Specifically one constructs a triangle in such a Riemannian manifold whose sides are minimizing geodesics and estimates one side in terms of the other two sides and the included angle. This is followed by a discussion of the `soul' of a Riemannian manifold. In Lemma 2.9 the authors do not explicitly construct the compact subset K but instead prove its existence by contradiction. In the proof of Corollary 2.11 the authors refer to the "ends" of a manifold before they introduce them in the next section, wherein they are defined in the usual way. Section 2.5 makes use of the fact, proved earlier, that the Busemann functions are superharmonic, in order to derive a `maximum principle' for connected Riemmanian manifolds. This in turn is used to prove the `splitting theorem', namely that a complete Riemannian manifold with non-negative Ricci curvature and two ends is isometric to a product of a compact manifold with the real numbers.The most important part of Chapter 2 concerns the notion of a `epsilon-neck structure' on a Riemannian manifold and the accompanying notion of an `epsilon-neck', which from the definition and the author's remarks is essentially an extended round cylinder. The authors show that a complete, positively curved Riemannian 3-manifold cannot contain an epsilon-neck of arbitrarily small scale (with scale being inversely related to the Ricci curvature). In the proof of this assertion it is not clear how they substantiate expression (2.2) in the proof of Proposition 2.19. Also, no concrete examples of epsilon-neck structures are given but the diagrams in the introduction assist in their understanding.The authors begin the study of Ricci flow in Chapter 3, viewing it first as a nonlinear version of the heat equation (for the Riemannian metric). This discussion is rather hurried, leaving the details of how to express the Ricci tensor in terms of local harmonic coordinates to the references. The use of harmonic coordinates guarantees that the Ricci tensor is only dependent on terms quadratic in the metric and its derivatives. In the discussion on shrinking solitons, the Lie derivative appears in expression (3.2) but is not introduced anywhere before then. The proof of local existence and uniqueness for Ricci manifolds is only sketched using the "DeTurck trick" which breaks the gauge invariance (under the diffeomorphism group) of the Ricci flow. The resulting flow, called the Ricci-DeTurck flow, is then strictly parabolic. A lot of space is then devoted to "index gymnastics" in the authors' discussion on the evolution of curvatures in an orthonormal frame. In addition, very detailed computations are given for how distances behave under Ricci flows and in estimation of the derivatives of the curvature under a Ricci flow. At this stage, the reader will have to take the authors word for the utility of these computations. The page space might have been used more productively to discuss epsilon-neck structures in greater detail than what is done so far in the book (with concrete examples given of these entities).The view of the Ricci flow as being essentially a heat equation for the time evolution of the metrics is continued in chapter 4, where the authors generalize the famous (strong) maximum principles for the heat equation to the case of Ricci flow: since the heat equation is parabolic, then positivity of its solution at the initial time guarantees that positivity for all times in the future. The authors prove the analog of this first for scalar curvature as a warm-up, and then prove the maximum principle for tensors. The gauge invariance of the Ricci flow equation is then broken in order to prove a maximum principle for the Ricci curvature. All of these considerations lead the authors to proving `strong' maximum principles for Riemann and Ricci curvature.The proof of Theorem 4.18 is not clear at first reading since the authors seem to merely restate the hypothesis of the theorem instead of assuming that the conclusion is not true and deriving a contradiction. They should have perhaps informed the reader right away that they intend to use continuity to derive a contradiction by assuming flatness for a choice of time less than the final time.

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